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A Computer Model of Ideological Confrontation between Two Political Forces in the Society

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Abstract. The paper provides an arbitrary dynamics model of social groups' political positions. These changes come because of the contacts between groups under the information influence of the external environment. Computer calculations on the model allow specifying the role of different parameters in the forecasts.

Keywords*:* social groups, socio-psychological attitude, imitative behavior, social interaction, Markov chains.

JEL Classification: C31.

INTRODUCTION

Social and political polarization in Russia during the last years is not weakening. The most part of population is involved in the active struggle for its political position. Mass media and mutual contacting in the population are contributing to these positions, often – against the real and profound interests of the people and the society as an entity. As a result because of the social and psychological law of imitation (similarity), the behavior of big masses may become stationary and irrational (Rashevsky, 1966).

The importance of scientific analysis of these processes and the development of the prognostic methods are beyond doubt. However, this analysis is set against the immense complexity of socium as an entity, which can be observed in the numerous characteristics and the specific connections in between (Helbing, 1994; Cioffi-Revilla, 2005, 2010; Opp, 2011). Often this complexity is aggravated by the well-founded qualitative theory having empirical and accepted outcomes. This happens because different branches of science, with their own methods and languages describe the social objects. That is why the design and analysis with mathematical and computer models, having the universal language, is unavoidable step for any social science (Makarov, 2013).

In this article, we analyze and use the mathematical models of inter-relations between the social groups of a single society, though varied in ideology or system of values for simulation computations. There are many research works on the movements and political confrontation topics. For example, there are articles on the political inequality between the states and the parties' political confrontation in the Senate etc. (Makarov, 2013; Baldassarri, Bearman, 2007).

Different methods are used for the model analysis. Agent-oriented models, permitting to analyze the interactions of a large number of the participants are in wide exploitation (Davern, 1997; Cioffi-Revilla, 2002; Wasserman, Faust, 1994; Makarov, Bakhtizin, 2013).

The Markov models are widely used in demography, the research of social mobility etc. (Staroverov, 1997; Semenchin, Babchenko, 2006). An alternative approach to the agent-oriented modeling is the design and interactions of the social groups, as well as modeling of these processes. Exactly here the Markov models and their modifications may be intensively used. The differential equations' apparatus open the new options for analyzing the dynamics of socio-economic processes (Moody, Douglas, 2003; Gavrilets, Anderson, Turchin, 2010; Weidlich, 2002).

The phenomenon of social networks and their possible influence on the social and political life arouse special interest and storm promises (Gubanov, Novikov, Chkhartishvilli, 2010; Baldassarri, Bearman, 2007).

Computer models and computation experiments are the only outcome, when it is impossible to get the adequate information on socium or when sociological experiments are too expensive. Computer modeling in these cases are considered the experiments, and the results of such quasi-experiments often bring useful and high-quality outcomes.

In the present article, we use differential equations, describing a behavior of a single man and Markov chains, describing the inter-group transactions to analyze inter-group interactions. Our main goal of computer modeling is the analysis of a process *per se* in time, as well as its final status, to which a system is driving.

SOCIAL SUBJECT BEHAVOIR MODEL

We estimate (Gavrilets, 1974) that three groups of factors determine behavior of any individual in a society, namely:

1) life *conditions* of an individual;

2) a system of *preferences and goals*;

3) individual *information* about the environment.

Some parameters of the environment and individual contacts are reflected in the following situations for analysis, a system of preferences is reflected in the guidelines, that is the political position of an individual, measured by real number. The information is reflected in the current ideas and guidelines about the other groups' aims and behavior. To change one's social status (choice of a group) is considered the aim of a particular behavior.

We shall show the use and options of the models, since our main model is based on the known models with Markov chains and differential equations.

Example 1. Use of differential equation. A classical model of simulation behavior was proposed by an outstanding Russian-American scientist N. Rashevsky[1]. He was one of the first founders of mathematical biology. We analyze N individuals every one of which may demonstrate one of the two types of behavior. The examples of such dichotomy may be political ("pro" or "contra"), religious (believers or nonbelievers), moral (chastity of "free love") etc. Choice of the one is determined by the purposes (settings) of an individual, measured by a real number ($-\infty < x < \infty$). The Gauss density determines the basic distribution за all the individuals by their purpose (before he gets information about the others' behavior)

$$N(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(\frac{-x^2}{2\sigma^2}\right),$$

where σ is a characteristic of scale of measurement of the purposes. Value x for every participant does not change in time. It is accepted that a probability of choosing the first type of behavior with x is given by function $p1(x)$, which is equal to probability integral with dispersion k^2 and zero mathematical expectation

$$\Phi(x,0,k) = \frac{1}{k\sqrt{2\pi}} \int_{-\infty}^{x} \exp\left(\frac{-u^2}{2k^2}\right) du. \tag{1}$$

Evidently, a probability of choosing the second type of behavior is $p2(x) = 1 - p1(x)$.

At any moment of time t the quantities $N1$ and $N2$ ($N1 + N2 = N$), realizing the first or the second behavior, are known to all the participants; son it turned to be possible to learn how much of those who had chosen the first type of behavior is bigger than those who had chose the second. This difference makes every one to deviate from his initial setting on value ψ. So, a choice of behavior 1 can be found from the general aim $y = x + \psi$, and

$$N1(\psi) = \int \Phi(x) p1(x+\psi) dx\,. \tag{2}$$

A value of "imitation behavior" ψ is the same (as compared to the initial aim x which remains unchanged with everybody) with everybody and is changing to the rule

$$\frac{d\psi}{dt} = A(N1 - N2) - a\psi\,, \tag{3}$$

where A is a coefficient of imitation (conformism), a is a coefficient (speed) of forgetting.

[1] N. Rashevsky was born in 1899 in Chernigov, studied in Kiev. In 1920 he emigrated form the Crimea with the White Army (Guard). Lived in Chicago after he competed lecturing in Turkey and Prague. Was a lectures in Chicago University, had some problems in McCarthy times.

Substituting $N1(\psi)$ and $N2(\psi)$ into equation (3) and analyzing its decision we can find, how the stationary conditions depend on the model parameters, and understand whether they would be stable or not. It is determined by the relation between the parameters A, a, σ, k, N.

The most important outcome from this model is an option to find a size of a "crowd" $N^* = 2AN / \sqrt{2\pi(\sigma^2 + k^2)}$, while exceeding which a "society" behaves irrational and a possibility to analyze a point of bifurcation when there can be an abrupt change of orientation of the majority of population.

Example 1. Use of Markov chain. Let us take the most simple model (Gavrilets, Ofman, 2012) of creating some discrete ordered setting (k = 1, …, n), for example, some "moral index" as a result of immediate contact of the individuals with the "outside environment. It is considered that a change of this index depends on immediate contact of individuals with "outside environment" that can be conditional "good" or conditional "bad".

Otherwise, as in the past, we have an individuals' distribution over the ordered values (1, ..., n) of the moral characteristics of the individuals: $X = (x_1, ..., x_n)$, $x_1 + ... + x_n = 1$. Here x_k is a share of individuals in the given aggregate, whose moral index is k.

At every moment of time $t = 0, 1, \ldots$ an individual with probability $P > 0$ meets bad environment, or with probability $P > 0$ — with good environment. At the same time $P + Q < 1$. Bad environment diminishes the moral index by one; it means that from condition k he moves to the condition $k - 1$. Good environment increases his moral index by one. The extreme (marginal) indexes' values do not change meeting (his) environment.

So, we have Markov chain with matrix of transitive probabilities **M**:

$$\mathbf{M} = \begin{pmatrix} 1-Q & Q & 0 & ... & 0 & 0 \\ P & 1-P-Q & Q & ... & 0 & 0 \\ 0 & P & 1-P-Q & ... & 0 & 0 \\ ... & ... & ... & ... & ... & ... \\ ... & ... & ... & ... & ... & .... \\ 0 & 0 & 0 & ... & P & 1-P \end{pmatrix}.$$

It is easy to confirm the truth of the following confirmation.

Statement. With any starting individuals' distribution over the moral index $X = (x_1, \ldots, x_n)$ the marginal distribution $X^* = (x_1^*, ..., x_n^*)$ forms a geometrical progression with *value* $q = Q / P$:

$$x_{k+1}^* = q x_k^*, \quad x_1^* = \frac{1}{1 + q + ... + q^{N-1}}. \tag{4}$$

Indeed, since matrix **M** is non-reducible, there is the single marginal condition X^*. Deciding a system N+1 of equations:

$$x_1^* + \ldots + x_n^* = 1, \quad (5)$$

$$X^* = X^* \mathbf{M}, \quad (6)$$

we get the correlations (4).

Let us note that progression X^* may be either increasing (with $Q > P$), or decreasing (with $P > Q$). The equality $P = Q$ stationary distribution will be uniform. This condition looks quite natural, because the better is one's life, the better people are around. The adverse statement may also be correct.

These examples illustrate the possible methods of mathematical interpretation of the dynamics in the social groups numbers due to the immediate contacts, as well as changes in the **aim**, that also changes when we observe the others' behavior (information influence).

THE CORRELATIONS IN THE BASIC MODEL

Before we turn to the description of correlations of the basic model, it is necessary to note that the author understands how to use a scheme of Markov chain further — as an approach to adequate mathematical description of social reality. There is no individual participants' behavior, they have no memory, there are no "absorbing" conditions, from which an individual cannot transit etc. These moments can be considered without formal difficulties, but the model dimension and the number of parameters will grow considerably.

Creation of social & political aim. Let us turn to the basic model. Let the society consist of N members, who form five social groups. Let us also take xx as a number of "active green" group; zz — as "active blue", x — as a group of supporting the "green", z — as a group of supporting the "blue", y — a number of "passive" part of society. So, at every moment we have balance in $y + x + z + xx + zz = N$.

Active groups of "green" and "blue" are recruited from the corresponding supporting groups in correlations with the aims: $e1$ — to activate the supporting "green", $e2$ — to activate the supporting "blue". Aims $e1$ and $e2$ give the functions of probabilities $q1(e1)$ and $q2(e2)$ to transit to the activities (and to the corresponding groups). So, at any moment t a number of activists increase by $q1(e1)x$ for the "greens" and $q2(e2)z$ for the "blues".

As for the vales of aims $e1$ and $e2$, they change concerning the number of supporting them, as well as of the mere aims $e1$ and $e2$ *per se*. The computation accounted the difference in aims (opinions, views) of every member from some analogous of medium aim of all the members in both groups. This difference depending on the sign (positive or negative) had led to increasing or decreasing the aim of every member.

The external standard influences the aim of a member form a group of supporters, where $E1$ and $E2$ are the fixed values of aim, imposed on the groups' members by the media (TV, radio, press etc.). Change in the tendency to be active in every supporting group depends on their relations inside the group and influence from the outside according to the formulas:

$$\Delta e1 = h\left[A1\left[\frac{1{,}5\, x_t\, e1_t + z_t(-e2_t)}{x_t + z_t} - e1_t \right] + B1(E1 - e1_t)\exp\left[-\left(\frac{E1 - e1_t}{12} \right)^2 \right] \right], \qquad (7a)$$

$$\Delta e2 = h\left[A2\left[\frac{-x_t\, e1_t + 1{,}7 z_t\, e2_t}{x_t + z_t} - e2_t \right] + B2(E2 - e2_t)\exp\left[-\left(\frac{E2 - e2_t}{13} \right)^2 \right] \right]. \qquad (7б)$$

We suppose, that the change in the aims of the tendencies to be active ($e1$, $e2$) in supporting groups is determined by a sum of two social & and psychological discomforts: the difference of the current aim from the analogous of weight average in both groups and its difference from the imposed from the outside standard ($E1$, $E2$).

The first augend (summand) is a linear function related to $e1$, $e2$, the second being non-linear (Figure 1). This non-linearity means that augmenting difference of the current aim *e* from the imposed standard *E* first grows, but later falls to the zero.

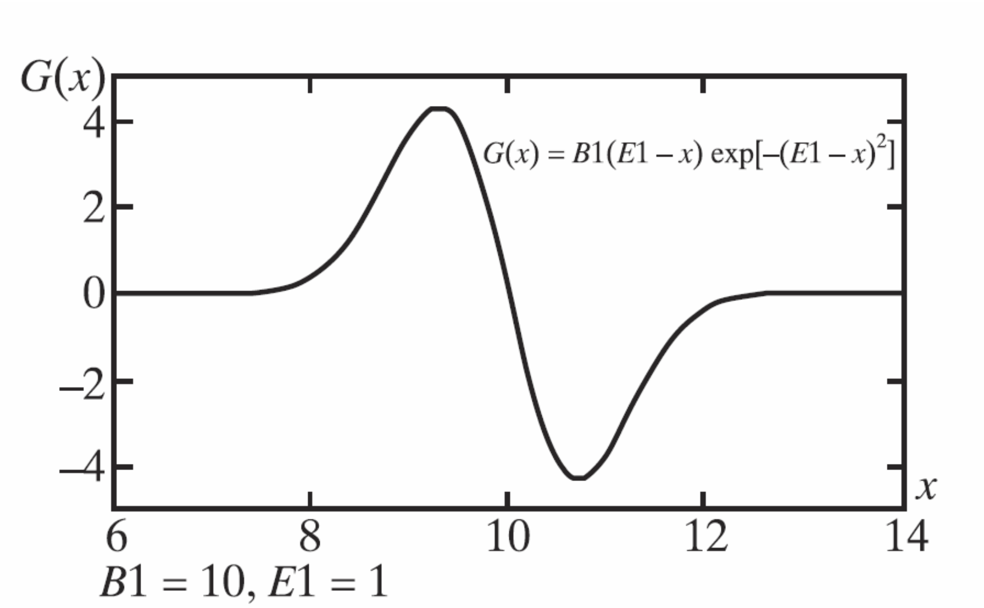

Figure 1

Some coefficients were introduced in computations for the formulas (7a)—(7b), though they have no significant meaning, but ease the visual interpretation of the computed dynamic trajectories.

Change in the aim ψ of the passive part of a society is described in the same way as Rashevsky equation (2), the outside influence *H* added. This equation defines the growth in the tendency of the passive part (in continuous ψ) at the expenses of the first type:

$$\frac{d\psi}{dt} = A(x + xx - z - zz) - a\psi + H. \qquad (7)$$

The solution of equation (7) was found in a series of iterations with several steps *h*. We have three differential equations, describing the patterns of creating the social position (*e*1, *e*2, ψ). We can see, that change in the aims is determined not only by its current values, but by the numbers of the active social groups.

Creating the number of group members. Social & ideological structure of society is vector of $W_t = (x_t, z_t, xx_t, zz_t, y_t)$, while its dynamics is described by Markov chain $W_{t+1} = W_t P_t$ (with probabilities of intra-group transitions of *p*(*x/y*) type), where

$$P_t = \begin{pmatrix} p_{x,x} & 0 & p_{x,xx} & 0 & \gamma 1 \\ 0 & p_{z,z} & p_{z,zz} & 0 & \gamma 2 \\ \rho & 0 & p_{xx,xx} & 0 & 0 \\ 0 & \mu & 0 & p_{zz,zz} & 0 \\ p_{y,x} & p_{y,z} & 0 & 0 & p_{y,y} \end{pmatrix}, \tag{8}$$

$$p_{y,xx} = p_{y,zz} = p_{x,z} = p_{z,x} = 0, \tag{8a}$$

$$p_{xx,y} = p_{zz,y} = p_{x,zz} = p_{z,xx} = 0, \tag{8б}$$

$$p_{xx,zz} = p_{xx,z} = p_{zz,x} = p_{zz,xx} = 0. \tag{8в}$$

Zero probabilities (8a)—(8b) show that transition is impossible, for example, — passive to the group of very active, though such a transition is possible in two steps. We suppose, that some transitions have constant probabilities, that means they do not depend on time and the values of the other variables. These are the probabilities of leaving the group of active (into the groups of supporting) and from the groups of supporting (to the passive part of socium):

$$p_{xx,x} = \rho,\ p_{zz,z} = \mu,\ p_{x,y} = \gamma_1,\ p_{z,y} = \gamma_2 .$$

Probabilities $p_{x,x}$, $p_{z,z}$, $p_{xx,xx}$, $p_{zz,zz}$, $p_{y,y}$ are determined as complements to one of the sum of all the rest probabilities of group transitions.

Creating the groups of maximalists takes place regularly depending on the current active mood of *e*1 or *e*2: in the groups of support

$$p_{x,xx} = \phi_1 \Phi(e1, 0, 1),\ p_{z,zz} = \phi_2 \Phi(e2, 0, 1), \tag{9}$$

where ϕ_1 and ϕ_2 are some coefficients, while function Φ is probability integral of type (1). As a result the active groups are formed according to the rule:

$$\varphi 1\,\mathrm{cnorm}(e1_t / 15{,}5) x_t + (1-\rho) xx_t = \omega \left[\mathrm{cnorm}(e2_t / 15) z_t + (1-\mu) zz_t\right] = z.$$

Some part of the active (ρ — for the ones and μ — for the others) continuously transit into the groups of support. Fragment γ1, γ2 in the groups of support return into the passive part of society. The passive part forms groups of support for the “blue” and the “green” according to Rishevsky model (σ^2, k^2 are the values of Gaussian dispersion, and ψ is the growth in favor of the first type of behavior). Positive parameters ξ_1, $\xi_2 < 1$ play the role of specifying coefficients for transition probabilities from the passive part of society into the groups of support:

$$p_{y,x} = \left[\frac{1}{\sqrt{2\pi(\sigma^2 + k^2)}} \int_{-\infty}^{\psi} \exp\left(\frac{-x^2}{2(\sigma^2 + k^2)} \right) dx \right] \xi_1 ,$$

$$p_{y,z} = \left[\frac{1}{\sqrt{2\pi(\sigma^2 + k^2)}} \int_{-\infty}^{\psi} \exp\left(\frac{-x^2}{2(\sigma^2 + k^2)} \right) dx \right] \xi_2 .$$

Given the values of parameters ξ_1, $\xi_2 < 1$ and formed the differential equations of the aim dynamics (ψ_t, $e1_t$, $e2_t$) and Markov relations for $(y_t, x_t, z_t, xx_t, zz_t)$, we have the united system of equations (7)—(8), specifying the dynamics of the population aims and the number of social groups; that means we can trace the trajectory of political contradictions in the society. Take a notice of correlation between the given parameters and the current values of all the variables, beginning from the present moment t, the solutions of differential equations and Markov chain (of every chain separately) may tend to their extreme (marginal) states.

These marginal (extreme) states mat be coherent and non-coherent. Therefore, if our Markov chain will always have the single stationary solution, then the differential equations would nave more that one stationary state, still more — some of them may be unstable (non-stationary). Unfortunately, we cannot propose a theoretical prove to the extreme (marginal) coherent state. But computer computations show stable algorithm convergence.

Computing the size of a step for changing the aims and the speed of changing the number of group members may be found in different procedures, because they specify model representations of the relative correlations between the velocities of reactions to the environment change. We may have the specific value of this step for every differential equation of the aims (notions).

Our model unites into on the dynamic system different mathematical objects: differential equations and Markov chains. It is evident, that vector of the states of the Markov sequence comes from the transition probabilities at *one*, *single* step. At the same time, we can randomly fix the size of this step through the consequent iterations inside the differential scheme (especially, if we would try to solve the differential equation), lest we define trajectory precisely enough. That is why the time period to which the transition probabilities relate should be related to the ticks of iteration reflecting the time in which the modeling change of the aims takes place, that depend on the Markov variables.

We may coordinate the time ticks, relating the transitive probabilities no to the fixed time gaps, but to the same ticks — as in the differential scheme. We may put the analogue of Markov chain $X_{t+1} = X_t P$ as $X_{t+1} = X_t + h(X_t P - X_t)$. At $h = 1$ we get the regular Markov chain. In our case it bring us to the same stationary state, but we shall be more careful in the interpretation of transition from one group into the other. Thus we additionally computed all the trajectories for this modification when the value of parameter *h* would be not 1, but 0,2. In the Attachment, we show the trajectories of this analogous of the basic model (see Figure A2).

MODEL COMPUTATIONS

We used the basic values of parameters for the primary computation. In Attachment (Figures A2 and A3) we showed some trajectories of the parameters, the values of basic variables and their extreme (marginal) states from Table 1. The Figure A2 shows the trajectories of all the group powers excluding the group supporting the "blues" (*z*). The Figure A3 shows the dynamics of all the three aims. We may see that in 200 — 300 ticks, a system comes into the balanced state, and the behavior trajectories may significantly differ. During the first ticks, many trajectories may be monotonous, but further they tend to the stationary state. However, the final balance does not depend on the random values of the initial state fro all the variants of our computations.

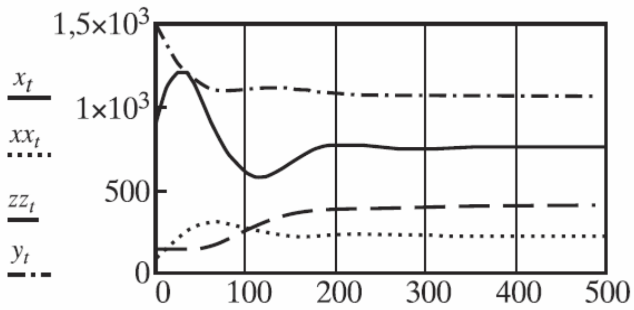

Figure 2

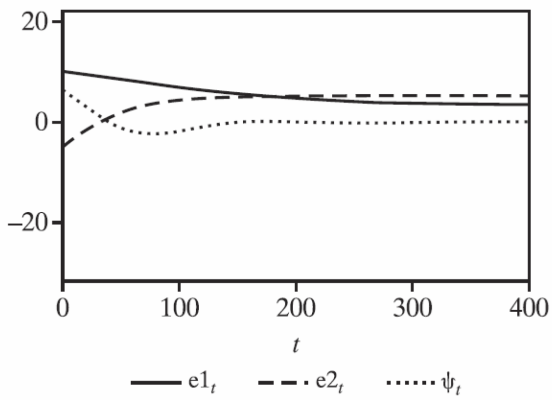

Figure 3

Analysis of structural model stability. The values if balanced (marginal) states did not depend on the random variations of the initial state, which are characteristics for the normal Markov chains and stable differential equations. However, this is not enough, when the values of model parameters are not defined precisely. Here comes a legitimate question: what deviations may be associated with the stationary values of the basic variables.

We have tested the stationary sensitivity of a system with the varying parameters of the basic model. We nominated random parameters in the neighborhood of the basic values for some variables using homogeneous and Gaussian distributions. Increasing the volumes of such random observations, we saw the gradual stabilization of the medium values (with increasing sample) of stationary (marginal, extreme) states. We observed the same stabilization of the related dispersions.

First we studied in turns the influence of the parameters *A* (simulation coefficient for the aim of the passive part of population), *H* (velocity regulation parameter of simulation aim change) and *k* (variety characteristics in behavior of the passive population) in the marginal (extreme) state.

With random values generator we modeled 2000 values of every one of the three parameters, and then computed 2000 related trajectories, approaching their extreme values. Random values were modeled using the Gaussian distribution with the zero expectations and standard deviation of 20% of the basic level of the parameter. Figure 4 shows some of these 2000 selected trajectories of parameter *y*. One can see, that they run higher and lower, but very close to the basic trajectory.

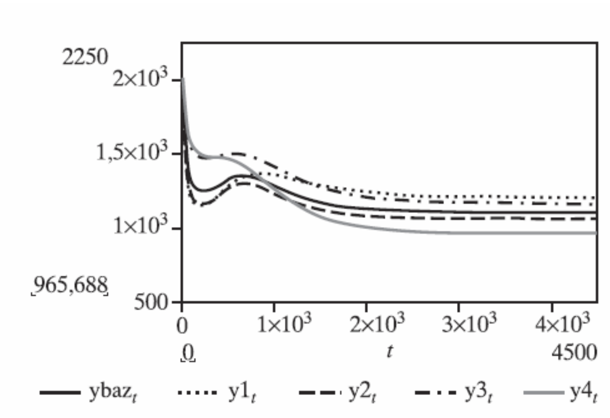

Figure 4

Computation show that selected medium of the extreme values of all the eight parameters do not differ significantly from each other as well as from their basic "stationary" value. The variety coefficients are in Table 1.

Table 1. Variety coefficients, %

	A	B	C	D
*e*1	1	0,607	0,341	1,3
*e*2	0.284	0.156	0.096	0.322
x	4.6	2.4	1.3	5.2
ψ	12.3	6.6	2.9	13.7
z	13.8	7.4	4.1	15.7
xx	4.8	2.5	1.4	5.4
zz	13.9	7.4	4.1	15.8
y	3.5	1.9	1.0	4.0

In the columns A, B, C and D we find variety coefficients of the model parameters *A*, *H* and *k*. Data in the Table 1 demonstrate that the values of "blue" numbers — *z* and *zz* — suffer the most individual fluctuations. Fluctuations of parameter "outside influence" *H* (on the simulation component ψ, the variation coefficient exceeding 13%) most influence these parameters.

The less sensitive to the activities of *e*1 and *e*2 are the parameters of the aims; the corresponding coefficients often do not exceed 1%. At last, the computations with the random sample of all three parameters increase the variations in the "stationary" state (column D), but the character of correlations between the dispersion of their parameters remains the same.

The same analysis was carried of simultaneous variation of the parameters' *A*, ξ1 and ξ2 aggregate; a number of random variations here was the same — 2000. In this case we used the Gaussian distribution and the homogeneous distribution with 20% interval from the basic. The mean stationary values of the variables for all 2000 realizations and variety coefficients are in Table 2.

Table 2. Variables, mean and mistakes

Variables			Gaussian distribution		Homogeneous distribution	
Variables	Initial	Stationary	Mean	Variation coefficients	Mean	Variation coefficients
*e*1	–11	4.231	4.231	0.011	4.23	0.009
*e*2	10	4.595	4.595	0.027	4.596	0.022
x	450	1524	1524	0.087	1520	0.072
ψ	–6	1.457	1.461	0.141	1.467	0.124
xx	350	176.334	174.592	0.128	175.759	0.108
z	75	185.213	185.248	0.088	184.757	0.073
zz	100	109.403	108.331	0.129	109.054	0.108
y	2125	1105	1108	0.125	1110	0.104

Calculations show, that mistakes in the values of the given "trinity" of parameters in their homogenous distribution gave weak influence the prognostic values as compared to the Gaussian distribution. Not only parameters ψ, *zz*, but also *xx* and *y* turned more sensitive to mistakes. Increased sensitivity of *y* looks unexpected; but we can explain this phenomenon by the fact that substitutions of parameters *H* and *k* for ξ1 and ξ2 in our "trinity" demonstrated more significance of transition form "passive" for the values of "passive" *per se*. We limited our research to the behavior of a system of five parameters. It is clear that the number of parameters may be easily increased.

CONCLUSIONS

First we remind the author's philosophical and methodological position.

1. Social processes are of regular type. We can treat this regularity as a tendency to a balanced state. The absence of any regularity — chaos — gives no opportunity for scientific description and prognosis.

2. This regularity allows its mathematical interpretation, meaning the objective laws of socium functioning behind. The main aim of our research was to computer analysis of mathematical model of a certain virtual reality, modeling the processes of ideological confrontations in a society. The basic scientific and sociological assumptions, to some extend obvious, of our research were the following.

1. Individual behavior depends on his/her inner social and psychological aim.

2. This aim is influenced by the contacts with the other individuals as well as from the mass-media.

3. The dominating aim in a society of a certain type or dominating number of individuals with a certain type of behavior attributes the strength of this influence.

4. We may give prognosis of individual behavior and aims after learning the influence of the aims and the information. The application of our model is quite pragmatic — behavior prognosis and control. Therefore, the **principle application of any model of this type** is behavior prognosis and control.

1. Our model demonstrate that group interactions in between and the society as an entity may be ideologically stable when the degree of aims' variations and numerical proportions between different ideological groups are stable, despite the continuous intergroup activities and information influence.

2. We can make prognosis of potentially stationary states (described by a number of members in social groups and their aims) using our model.

3. Our model and the proposed method of its analysis allows analyzing the influence on different model parameters reflecting socium characteristics, or the abilities to control the processes in concern.

4. Despite the virtual character of sociun in a model, its parameters may be attributed statistical values or expert weights, some of the parameters — easily and the others — with some difficulties.

5. Our model allows making simplifications, amendments, additions and modifications. In particular, some parameters may be taken as controlling; varying them, one may influence the modeling behavior of the groups. These parameters may be H, $E1$, $E2$, $B1$, $B2$ etc.

6. Ony may add the influence of outside environment, for example, level of well-being (or criminality) on parameter $B1$ ($A2$). We can introduce the adverse influence of maximalists' number

on the processes or on the outside environment. As a result, the whole problem is in the basic data. Getting this data is a very special and complex problem.

ATTACHMENT

1. Value of the basic parameters

*A*1=0,2; *A*2=0,16; *A*=0,0013; *a*=0,1062; *B*1=0,85; *B*2=3; *E*1=4; *E*2=5; α1=0,001; α2=0,001; β1=0,001; β2=0,0011; δ1=0,018; δ2=0,016; ρ=0,02; μ=0,01; σ=1,4; *h*=0,1; *h*1=0,01; γ1=0,02; γ2=0,03; *k*=2; φ1=0,01; φ2=0,001; ω=0,01; ξ1=0,03; ξ2=0,034,

2. Parameters' dynamics of the basic parameters

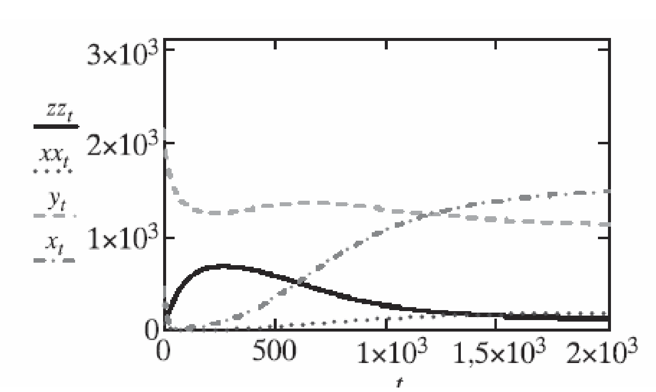

Figure A1

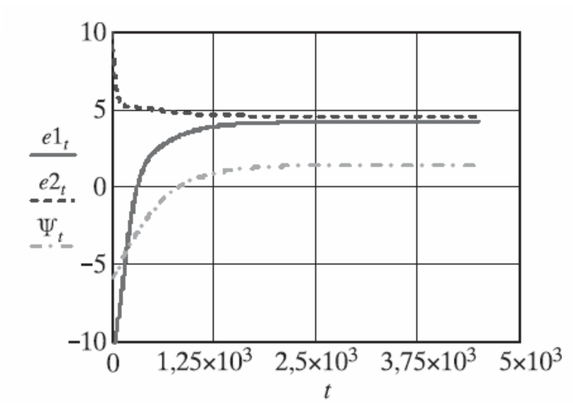

Figure A2

3. Parameters' dynamics of modified Markov chain

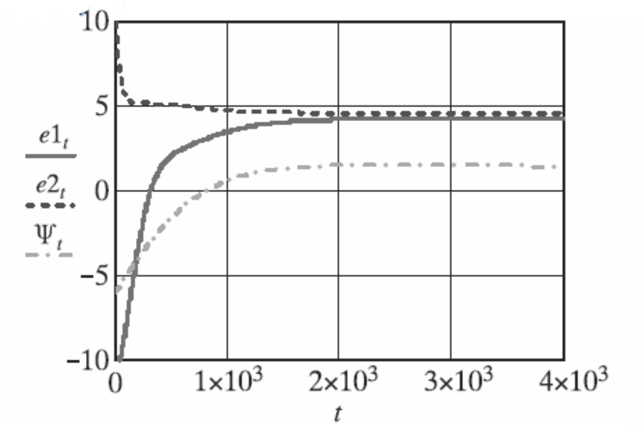

Figure A3

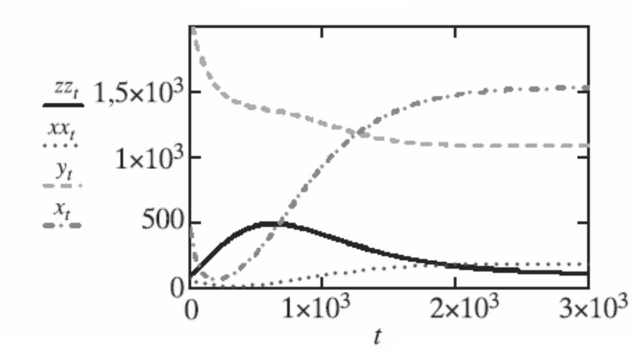

Figure A4

4. Mean stationary values of *zz*-parameter for 150*h* observations

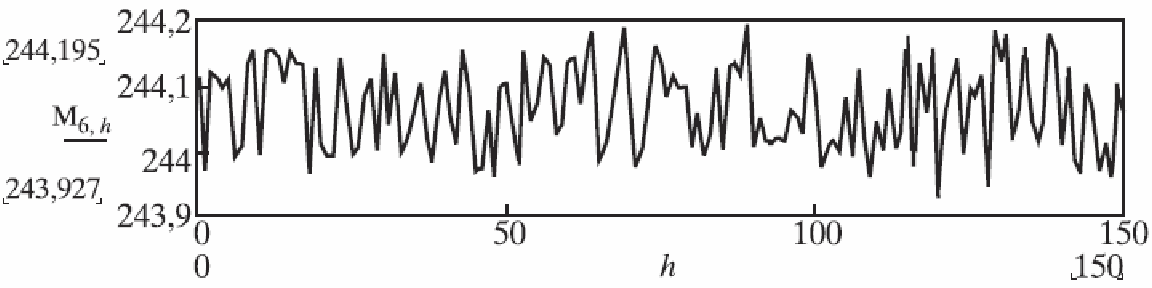

Figure A5

5. Histograms for variables *x* and *e*1 for 2000 observations with the homogenous mistakes' distribution of the parameters

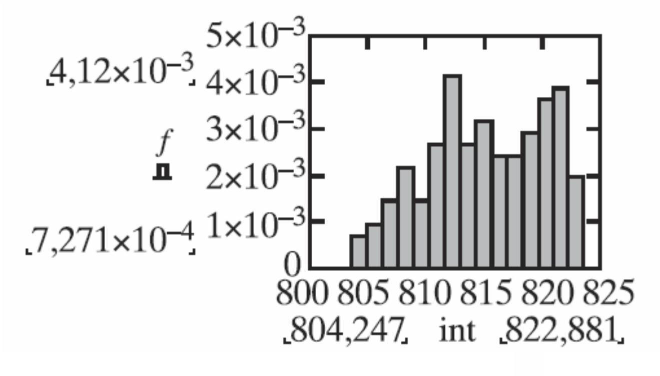

Figure A6

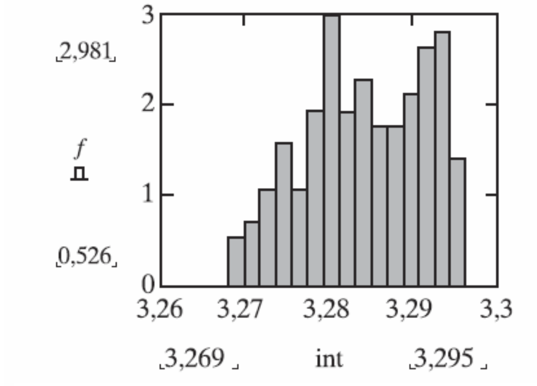

Figure A7

6. The graph of selected mean values and dispersions for parameter ψ concerning the amount of a sample

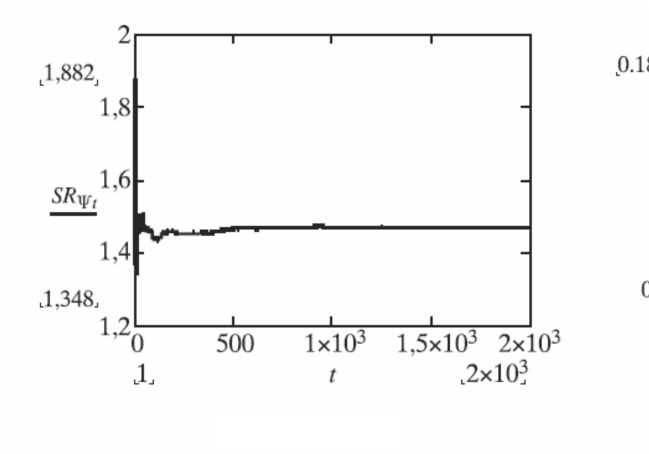

Figure A8

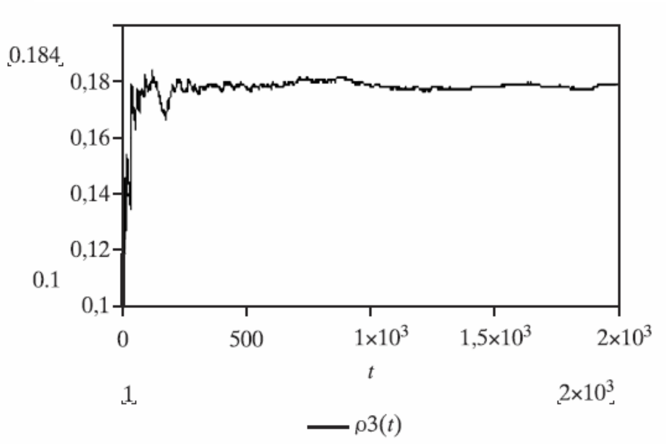

Figure A9

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